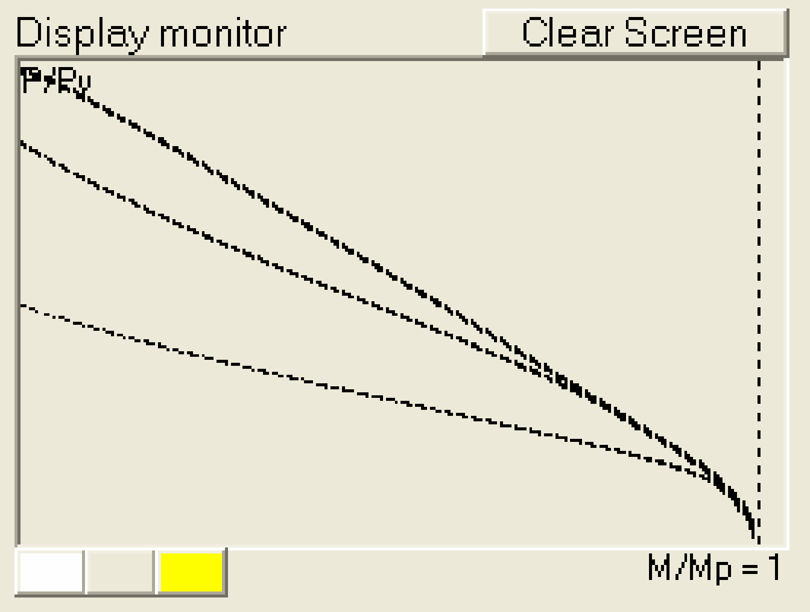
Moment - Axial Force Interaction Report and Program



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THINK GLOBAL, ACT LOCAL

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1. Scope

Calculate the moment – axial force interaction curve for a hinged column with variable slenderness (λ), initial crooked $w_0(x) = 0.001l \sin\frac{\pi x}{l}$. The end moments are M_0 and βM_0 .

Concepts:

a. Column section profile is IPB.
b. Choose capacity curve from Chen of nethercot curves.
c. Calculate every point of the interaction curve according to the contact point of capacity curve and elastic M-P curve.

2. Moment equation

At first we should find the column curvature differential equation and then solve it to find the moment distribution along the column (member). So we draw a schematic view of column end conditions, curvature and loadings direction Fig 2.1.

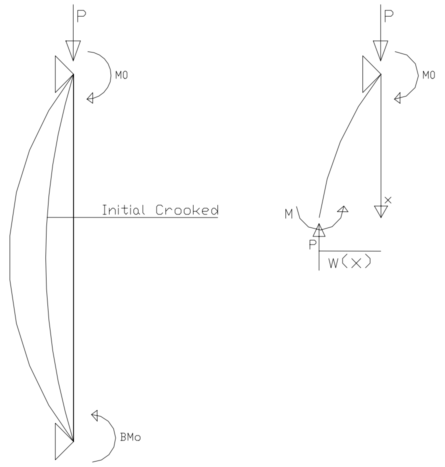


Fig 2.1 Column diagram

$$M(x) = Pw + M_0 + \frac{(\beta - 1)M_0}{l}x \qquad (1)$$

But we know: $\begin{cases} M = -EIW''_{total} \\ W''_{total} = w'' - w''_0 \end{cases} \Rightarrow M = -EI(w'' - w''_0) \qquad (2)$

(1) & (2) ➔ $-EIw'' + EIw''_0 = Pw + M_0 + \frac{(\beta - 1)M_0}{l}x$

$$EIw'' + Pw = EIw''_0 - M_0 - \frac{(\beta - 1)M_0}{l}x$$

$$\frac{P}{EI} = \mu^2 \Rightarrow w''(x) + \mu^2 w(x) = w''_0(x) - \frac{M_0}{EI} - \frac{(\beta - 1)M_0}{EIl}x \qquad (3)$$

$$w_0(x) = 0.001l \sin\frac{\pi x}{l}$$

$$w'_0(x) = 0.001\pi \cos\frac{\pi x}{l}$$

$$w''_0(x) = -\frac{0.001\pi^2}{l}\sin\frac{\pi x}{l} \qquad (4)$$

(3) & (4) ➔ $w''(x) + \mu^2 w(x) = -\frac{0.001\pi^2}{l}\sin\frac{\pi x}{l} - \frac{M_0}{EI} - \frac{(\beta - 1)M_0}{EIl}x$

Solving the above equation:

a. Solving the homogeneous equation

$$w''(x) + \mu^2 w(x) = 0 \Rightarrow w_h(x) = c_1 \sin \mu x + c_2 \cos \mu x$$

b. solving the no homogeneous equation:

Part I: $w''(x) + \mu^2 w(x) = -\frac{0.001\pi^2}{l}\sin\frac{\pi x}{l}$

$$\Rightarrow w_{P1}(x) = c_3 \sin\frac{\pi x}{l} + c_4 \cos\frac{\pi x}{l}$$

Part II: $w''(x) + \mu^2 w(x) = -\frac{M_0}{EI}$

$$\Rightarrow w_{P2}(x) = c_5$$

Part III: $$w''(x)+\mu^2 w(x)=-\frac{(\beta-1)M_0}{EIl}x$$

$$\Rightarrow w_{P3}(x)=c_6x+c_7$$

Assigning these three parts of answer by super position principle:

$$\Rightarrow w_P(x)=c_3\sin\frac{\pi x}{l}+c_4\cos\frac{\pi x}{l}+c_5+c_6x+c_7$$

$$w_P(x)=c_3\sin\frac{\pi x}{l}+c_4\cos\frac{\pi x}{l}+c_6x+(c_5+c_7)$$

$$w'_P(x)=c_3\frac{\pi}{l}\cos\frac{\pi x}{l}-c_4\frac{\pi}{l}\sin\frac{\pi x}{l}+c_6$$

$$w''_P(x)=-c_3\frac{\pi^2}{l^2}\sin\frac{\pi x}{l}-c_4\frac{\pi^2}{l^2}\cos\frac{\pi x}{l}$$

Then we put the results in differential equation:

$$w''(x)+\mu^2 w(x)=-\frac{0.001\pi^2}{l}\sin\frac{\pi x}{l}-\frac{M_0}{EI}-\frac{(\beta-1)M_0}{EIl}x=$$

$$-c_3\frac{\pi^2}{l^2}\sin\frac{\pi x}{l}-c_4\frac{\pi^2}{l^2}\cos\frac{\pi x}{l}+c_3\mu^2\sin\frac{\pi x}{l}+c_4\mu^2\cos\frac{\pi x}{l}+c_6\mu^2x+\mu^2(c_5+c_7)$$

And the constants (C3 ~ C7) will be

1) $$\mu^2(c_5+c_7)=-\frac{M_0}{EI}\Rightarrow(c_5+c_7)=-\frac{M_0}{\mu^2EI}$$

2) $$-\frac{(\beta-1)M_0}{EIl}x=c_6\mu^2x\Rightarrow c_6=-\frac{(\beta-1)M_0}{\mu^2EIl}$$

3) $$-c_4\frac{\pi^2}{l^2}\cos\frac{\pi x}{l}+c_4\mu^2\cos\frac{\pi x}{l}=0\Rightarrow c_4=0$$

4) $$-c_3\frac{\pi^2}{l^2}\sin\frac{\pi x}{l}+c_3\mu^2\sin\frac{\pi x}{l}=-\frac{0.001\pi^2}{l}\sin\frac{\pi x}{l}$$

$$c_3(\mu^2-\frac{\pi^2}{l^2})=-\frac{0.001\pi^2}{l}\Rightarrow c_3=\frac{-\dfrac{0.001\pi^2}{l}}{\mu^2-\dfrac{\pi^2}{l^2}}$$

$$\Rightarrow c_3=\frac{\dfrac{\pi^2}{l^2}(0.001l)}{\dfrac{\pi^2}{l^2}-\mu^2}=\frac{1}{1-\dfrac{P}{P_E}}(0.001l)$$

Then we assign these constants in particular answer:

$$w_p(x)=\frac{1}{1-\dfrac{P}{P_E}}0.001l\sin\frac{\pi x}{l}-\frac{(\beta-1)M_0}{EIl\mu^2}x-\frac{M_0}{EI\mu^2}$$

The general solution are equal to summation of homogeneous and no homogeneous answers :

$$w(x)=w_h(x)+w_p(x)$$

$$\Rightarrow w(x)=c_1\sin\mu x+c_2\cos\mu x+\frac{1}{1-\dfrac{P}{P_E}}0.001l\sin\frac{\pi x}{l}-\frac{(\beta-1)M_0}{EIl\mu^2}x-\frac{M_0}{EI\mu^2}$$

The constant values of C1 and C2 will be calculated by assigning the boundary conditions in the equation:

$$\begin{cases} w(x=0)=0\Rightarrow c_2-\dfrac{M_0}{EI\mu^2}=0 \\ w(x=l)=0\Rightarrow c_1\sin\mu l+\dfrac{M_0}{EI\mu^2}\cos\mu l-\dfrac{(\beta-1)M_0}{EI\mu^2}-\dfrac{M_0}{EI\mu^2}=0 \end{cases}$$

$$\Rightarrow\begin{cases} c_2=+\dfrac{M_0}{EI\mu^2} \\ c_1=-\dfrac{M_0}{EI\mu^2}\dfrac{\cos\mu l}{\sin\mu l}+\dfrac{(\beta-1)M_0}{EI\mu^2\sin\mu l}+\dfrac{M_0}{EI\mu^2\sin\mu l} \end{cases}$$

Then the total displacement of the column will be:

$$\Rightarrow w(x)=\left[-\frac{M_0}{EI\mu^2}\frac{\cos\mu l}{\sin\mu l}+\frac{(\beta-1)M_0}{EI\mu^2\sin\mu l}+\frac{M_0}{EI\mu^2\sin\mu l}\right]\sin\mu x+\frac{M_0}{EI\mu^2}\cos\mu x+$$

$$+\frac{1}{1-\dfrac{P}{P_E}}0.001l\sin\frac{\pi x}{l}-\frac{(\beta-1)M_0}{EIl\mu^2}x-\frac{M_0}{EI\mu^2}$$

$$\Rightarrow w'(x) = \left[-\frac{M_0}{EI\mu}\frac{\cos\mu l}{\sin\mu l} + \frac{(\beta-1)M_0}{EI\mu\sin\mu l} + \frac{M_0}{EI\mu\sin\mu l} \right]\cos\mu x - \frac{M_0}{EI\mu}\sin\mu x +$$

$$+\frac{\frac{\pi}{l}}{1-\frac{P}{P_E}}0.001l\cos\frac{\pi x}{l} - \frac{(\beta-1)M_0}{EIl\mu^2}$$

The total curvature:

$$\Rightarrow w''(x) = -\left[-\frac{M_0}{EI}\frac{\cos\mu l}{\sin\mu l} + \frac{(\beta-1)M_0}{EI\sin\mu l} + \frac{M_0}{EI\sin\mu l} \right]\sin\mu x - \frac{M_0}{EI}\cos\mu x - \frac{\frac{\pi^2}{l^2}}{1-\frac{P}{P_E}}0.001l\sin\frac{\pi x}{l}$$

And now we can find the moment in each section (x) according to the curvature of the column.

$$M(x) = -EI(w''(x) - w_0''(x))$$

$$M(x) = \left[-M_0\frac{\cos\mu l}{\sin\mu l} + \frac{(\beta-1)M_0}{\sin\mu l} + \frac{M_0}{\sin\mu l} \right]\sin\mu x + M_0\cos\mu x + \frac{EI\frac{\pi^2}{l^2}}{1-\frac{P}{P_E}}0.001l\sin\frac{\pi x}{l}$$

$$-\frac{EI\pi^2}{l^2}0.001l\sin\frac{\pi x}{l}$$

$$M(x) = M_0\frac{(\beta-\cos\mu l)}{\sin\mu l}\sin\mu x + M_0\cos\mu x + \frac{P}{1-\frac{P}{P_E}}0.001l\sin\frac{\pi x}{l}$$

This moment equation will be used to find the moment distribution along the column.

3. Capacity curves

Chen and Nethercot suggested two capacity equations:

3.1. Chen 1971

Chen suggested equation (3-1) where a_1, a_2 and a_3 depend on the column section profile and also the numbers of the profile, but the average values are listed below:

$$\frac{M}{M_P} + a_1\left(\frac{P}{P_Y}\right) + a_2\left(\frac{P}{P_Y}\right)^2 = a_3 \qquad (3\text{-}1)$$

Wide flange profiles bent on strong axe:

Condition	a_1	a_2	a_3
$0 < \frac{P}{P_Y} \le 0.225$	0	2.378	1
$2.225 < \frac{P}{P_Y} \le 1$	1.03	0.085	1.115

Wide profiles bent on weak axe:

Condition	a_1	a_2	a_3
$0 < \frac{P}{P_Y} \le 0.225$	0	0.185	1
$2.225 < \frac{P}{P_Y} \le 1$	-0.821	1.709	0.888

3.2. Nethercot 1989

Nethercot suggested another form of capacity curves that just differ in coefficients. The average values are listed bellow:

Wide profiles, bent on strong axe:

Condition	a_1	a_2	a_3
$0 < \frac{P}{P_Y} \le 0.2$	0	2.5	1
$2.2 < \frac{P}{P_Y} \le 1$	1.125	0	1.125

Wide profiles, bent on weak axe:

Condition	a_1	a_2	a_3
$0 < \frac{P}{P_Y} \leq 0.447$	0	0.5	1
$447 < \frac{P}{P_Y} \leq 1$	0	1.125	1.125

Note that in this curves the capacity of the section is important and we don't care about buckling because buckling occurs in the member not section.

4. Elastic M-P curve

The moment equation calculated in section 2 will be used to find the maximum moment in member depend on a specified end moment (M_0) with varying P from 0 to P_y. Because the M_0 changes between 0 and M_y, if the results be drawn it will call Elastic M-P curve as the moment are not greater than the yield moment.

The algorithm for this procedure is simple, just in two loops, the first for P variations and the inner loop for x variations along the member from 0 to L (length of the member). Further information on this algorithm will be given in next sections.

5. Interaction curve

Interaction curve will be made by the M_0/Mp and P/Py depends on the contacting point of Capacity curve and Elastic M-P curves. Results of the calculated curves are illustrated in section 9.

6. calculation algorithm

Chart 6-1 illustrates the algorithm diagram of the interaction procedure. The initial values for M and P will be very small such 0.001 and for x will be zero.

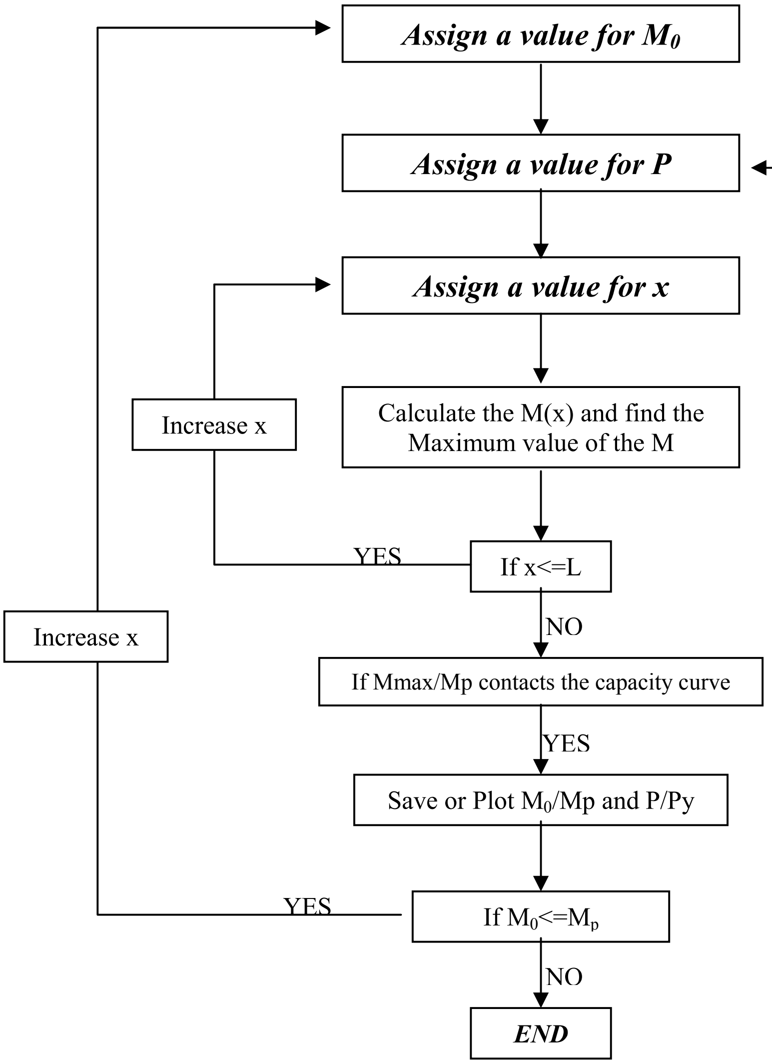


Chart 6.1 calculation algorithm

7. Interaction program source

The language of this program is Visual Basic, and because this program can calculate different materials, sections, loading conditions and different capacity curves it may be seem complicated, but the main segment for calculation is simple. Here are the comments for different segments.

7.1. Global variables definition

In this segment the global variables defined and the comments for every variable are in the bellow.

```
Section Properties
Dim Area As Single            Section Area
Dim Ix As Double              Section Moment of inertia
Dim Sx As Single              Section Module
Dim rx As Single  '           Gyration
Dim sngSlender As Single      Slenderness

Material Properties
Dim intZigma As Integer       Yield Stress
Dim E As Double               elastic modulus

End moment ratio
Dim Beta As Double             end moment ratio

Constant PI = 3.14
Dim PI As Double               number of Pi = 3.14

Capacity Curve Parameters depend on what method selected (Chen pr Nether
cot)
Dim CapacityA1 As Single      a1 for P/Py < 0.225
Dim CapacityA2 As Single      a2
Dim CapacityA3 As Single      a3
Dim CapacityA4 As Single      a1 for P/Py > 0.225
Dim CapacityA5 As Single      a2
Dim CapacityA6 As Single      a3
Dim CapacityLimit As Single   depend on the method the point of slope changing
                              0.225 for Chen and 0.2 for Nethercot

' calculation parameters for increasing steps
Dim Pincrease As Double       axial force increasing step by : Pincrease * Py
Dim Mincrease As Double       Moment increase step by : Mincrease * Mp
Dim xIncrease As Double      X increase for calculation maximum of moment :
                              xincrease* length
```

7.2. Information collecting

After clicking the “Go” bottom the information of the form will be gathered by a procedure called data. There is no calculation is this procedure and just collect the data from the form and assign the values to variables according the selected methods and values.

```
Sub data()

Selecting the Section properties
Select Case List1.ListIndex
    Case 0                          IPB 100
            Area = 26
            Ix = 450
            Sx = 89.9
            rx = 4.16
    Case 1                          IPB 120
            Area = 34
            Ix = 864
            Sx = 144
            rx = 5.04
   Case 2                           IPB 140
            Area = 43
            Ix = 1510
            Sx = 216
            rx = 5.93
   Case 3                           IPB 160
            Area = 54.3
            Ix = 2490
            Sx = 311
            rx = 6.78
   Case 4                           IPB 180
            Area = 65.3
            Ix = 3830
            Sx = 426
            rx = 7.66
   Case 5                           IPB 200
            Area = 78.1
            Ix = 5700
            Sx = 570
            rx = 8.54
   Case 6                           IPB 220
            Area = 91
            Ix = 8090
            Sx = 736
            rx = 9.43
   Case 7                           IPB 240
            Area = 106
            Ix = 11260
            Sx = 938
            rx = 10.3
   Case 8                           IPB 260
            Area = 118
            Ix = 14920
            Sx = 1150
            rx = 11.2
```

```
      Case 9                         IPB 280
        Area = 131
        Ix = 19270
        Sx = 1380
        rx = 12.1
        Case 10                      IPB 300
        Area = 149
        Ix = 25170
        Sx = 1680
        rx = 13
End Select

selecting the slenderness of column
  sngSlender = CSng(Text4.Text)
Yield stress
   intZigma = CInt(Text2.Text)
Elastic Modulus
   E = CDbl(Text3.Text)
Reading Beta from form
   Beta = CDbl(Text1.Text)
choosing the Capacity Curvetur
If Option1.Value = True Then          CHEN
        CapacityA1 = 0
        CapacityA2 = 2.378
        CapacityA3 = 1
        CapacityA4 = 1.03
        CapacityA5 = 0.085
        CapacityA6 = 1.115
        CapacityLimit = 0.225
Else                                 Nether cot
        CapacityA1 = 0
        CapacityA2 = 2.5
        CapacityA3 = 1
        CapacityA4 = 1.125
        CapacityA5 = 0
        CapacityA6 = 1.125
        CapacityLimit = 0.2
End If

calculation parameters
Pincrease = CDbl(Text7.Text)
Mincrease = CDbl(Text8.Text)
xIncrease = CDbl(Text9.Text)

End Sub
```

7.3. Calculation

This is the main procedure of the program, the structure are similar to the calculation algorithm.

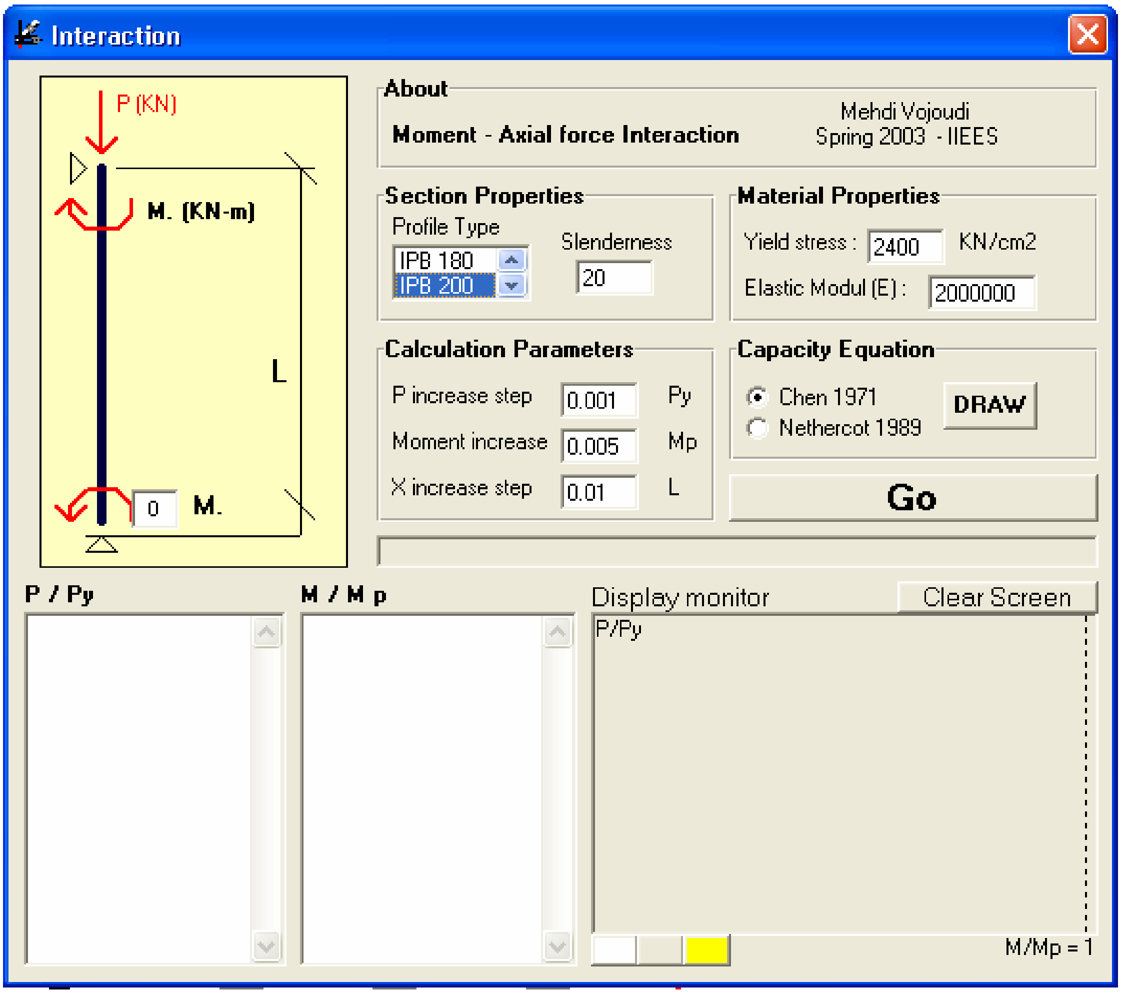
Go bottom procedure:

```
Private Sub Command1_Click()
Call data ' collect the information from the form

Definition the local variables
Dim Length As Double     length of the column
Dim Moo As Double        μ^2 = P/EI
Dim Py As Double         Yield Axial Force = Yield Stress (σ) * Area of section (Area)
Dim My As Double         Yield Moment     = Yield Stress (σ) * Module of section (Sx)
Dim Mp As Double         Plastic Moment   = 1.15 * My

Dim P As Double              variable for axial force
Dim M As Double              variable for M0
Dim x As Double              variable for length of the column
Dim Moment As Double         variable for moment in the x position
Dim Mmax As Double           Maximum Moment of column
Dim Pe As Double             Euler Force

Dim strp As String           Output string that contains the P/Py values
Dim strm As String           Output string that contains the M/Mp values
Dim a As Double              An auxiliary variable
Dim b As Double              An auxiliary variable

Setting the variables value
PI = 3.14
Length = sngSlender * rx     calculating the length of the column
Py = intZigma * Area
Pe = (PI ^ 2) * E * Ix / Length / Length
My = intZigma * Sx
Mp = 1.15 * My
strp = ""
strm = ""
Picture1.DrawWidth = 1 sets the display monitor properties

Start of calculation of interaction curves
M = 0.0001                   initial value for M0
------------First Loop-----------
Do While (M <= 1.15 * My)
     P = 0.00001   Initial value for P
-----------Second loop-----------
       Do While (P <= Py)
            Moo = (P / E / Ix) ^ 0.5
            x = 0.005 * Length
           Mmax = 0
-----------Third Loop-----------
              Do While (x <= Length)
                 Moment = (Beta - Cos(Moo * Length)) * M * Sin(Moo * x) / Sin(Moo *
                 Length) + M * Cos(Moo * x) + (P / (1 - P / Pe)) * 0.001 * Length *
                 Sin(PI * x / Length)

       If (Moment > Mp) Then GoTo 10 Checks not to do unnecessary calculation

               If (Mmax < Moment) Then
                          Mmax = Moment

                End If
             x = x + xIncrease * Length              Increasing x
             Loop
```

```
Compare the Mmax with Capacity curve (Chen or nether cot and
            If (P / Py) < CapacityLimit Then
                  If (Abs((Mmax / My) - 1.15 * (CapacityA3 - CapacityA1 * (P / Py) -
                  CapacityA2 * (P / Py) ^ 2)) < 0.005) Then
                  Picture1.PSet (M / My / 1.15 * Picture1.Width * 0.95, (1 - P / Py) *
Picture1.Height), 0

                   a = P / Py
                   b = M / My / 1.15
                    strp = strp + CStr(a) + Chr(13) + Chr(10)
                    strm = strm + CStr(b) + Chr(13) + Chr(10)

                 End If
           Else
                If (Abs((Mmax / My) - 1.15 * (CapacityA6 - CapacityA4 * (P / Py) -
CapacityA5 * (P / Py) ^ 2)) < 0.005) Then
                    Picture1.PSet (M / My / 1.15 * Picture1.Width * 0.95, (1 - P / Py) *
Picture1.Height), 0
                    a = P / Py
                   b = M / My / 1.15
                    strp = strp + CStr(a) + Chr(13) + Chr(10)
                    strm = strm + CStr(b) + Chr(13) + Chr(10)
                 End If
            End If

          P = P + Pincrease * Py              Increasing P
       Loop
10
M = M + Mincrease * Mp      increasing M0
ProgressBar1.Value = (M / Mp) * 100
Loop

Text5.Text = strp     Write the out put
Text6.Text = strm     Write the out put
End Sub
```

8. How to use Program

Interaction program is simple to use and the form is designed user friendly. The different parts of information divided, the best and common values are default and can be changed. (Fig 8.1)



About

Moment - Axial force Interaction

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Spring 2003 - IIEES

Section Properties

Profile Type: IPB 180, IPB 200

Slenderness: 20

Material Properties

Yield stress : 2400 KN/cm2

Elastic Modul (E) : 2000000

Calculation Parameters

P increase step 0.001 Py

Moment increase 0.005 Mp

X increase step 0.01 L

Capacity Equation

● Chen 1971
○ Nethercot 1989

0 M.

Fig 8.1 Program appearance

8.1 Schematic diagram

This is a schematic diagram for loading condition and end hinges. The β variable can be entered here.

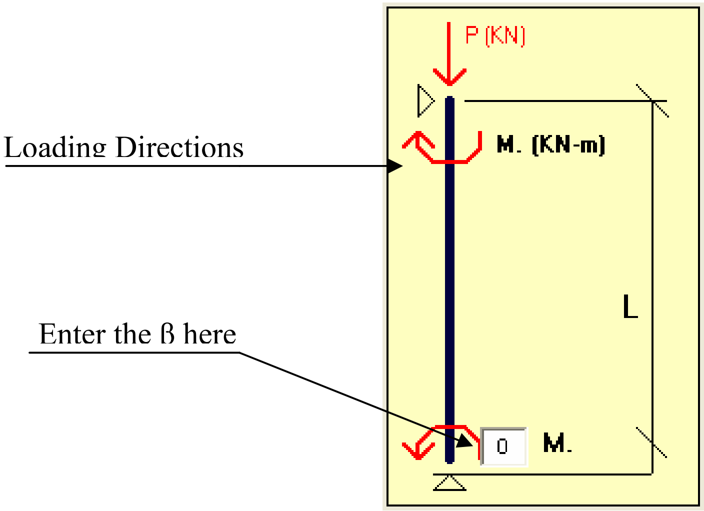


Fig 8.2 schematic diagram of loading conditions

8.2 Section properties

The section size and slenderness can be selected here, fig 8.3.

Section Properties

Profile Type: IPB 180, IPB 200

Slenderness: 20

Figs 8.3 select the Profile size and slenderness

8.3 Material properties

You can enter the material properties such as yield stress and elastic module it the specified places, Fig 8.4.

Material Properties

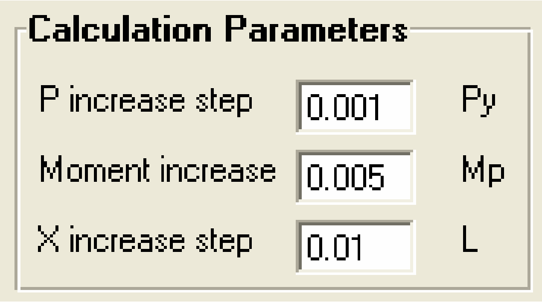
Yield stress : 2400 KN/cm2

Elastic Modul (E) : 2000000

Fig 8.4 Material properties

8.4 Calculation parameters

The increasing steps for M_0, P and x can be defined and changed here. Note that the steps influence the accuracy of the results.



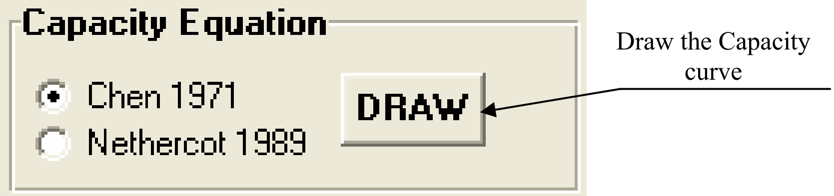
Calculation Parameters

P increase step	0.001	Py
Moment increase	0.005	Mp
X increase step	0.01	L

Fig 8.5 increasing steps

8.5 Capacity equation

The capacity equation can be selected here to be used in calculations. The selectable equations are Chen 1971 and Nether cot 1989. The capacity curve will be plotted by clicking the DRAW bottom.



Capacity Equation

● Chen 1971

○ Nethercot 1989

Draw the Capacity curve

Fig 8.6 Capacity equation selection

8.6 Go bottom

This bottom, starts the calculation of the interaction curve and by calculating each point, it will be plotted on the display monitor.

Fig 8.7 Go bottom to start the calculation

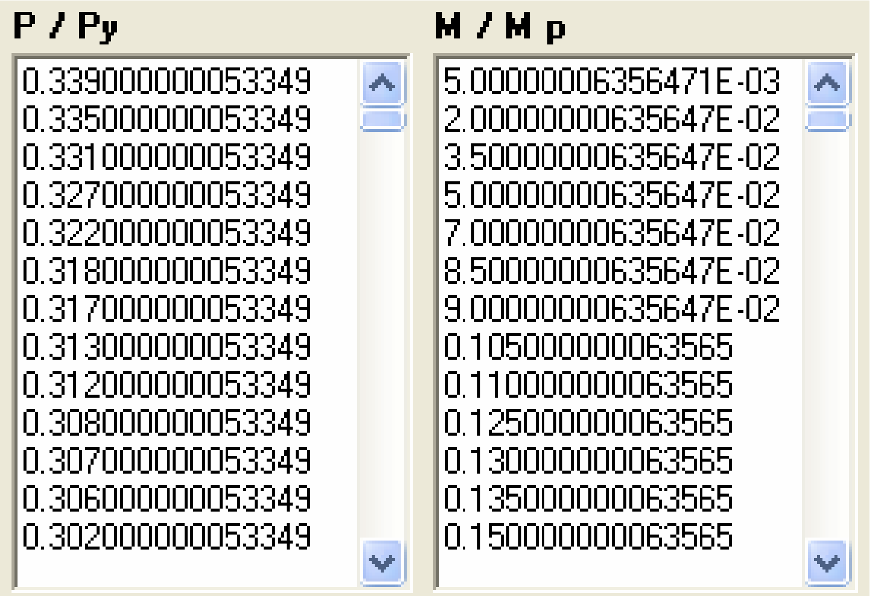
8.7 Progress bar

During the calculation, this will show the progress.

Fig 8.8 Progress bar

8.8 Results

There are two columns that show the results of calculation for P/Py and M/Mp that can be easily selected or printed.



P / Py	M / M p
0.339000000053349	5.00000006356471E-03
0.335000000053349	2.0000000635647E-02
0.331000000053349	3.5000000635647E-02
0.327000000053349	5.0000000635647E-02
0.322000000053349	7.0000000635647E-02
0.318000000053349	8.5000000635647E-02
0.317000000053349	9.0000000635647E-02
0.313000000053349	0.105000000063565
0.312000000053349	0.110000000063565
0.308000000053349	0.125000000063565
0.307000000053349	0.130000000063565
0.306000000053349	0.135000000063565
0.302000000053349	0.150000000063565

Fig 8.9 Results

8.9 Display monitor

Displays a schematic plot of results, It can be cleared and the back color can be changed

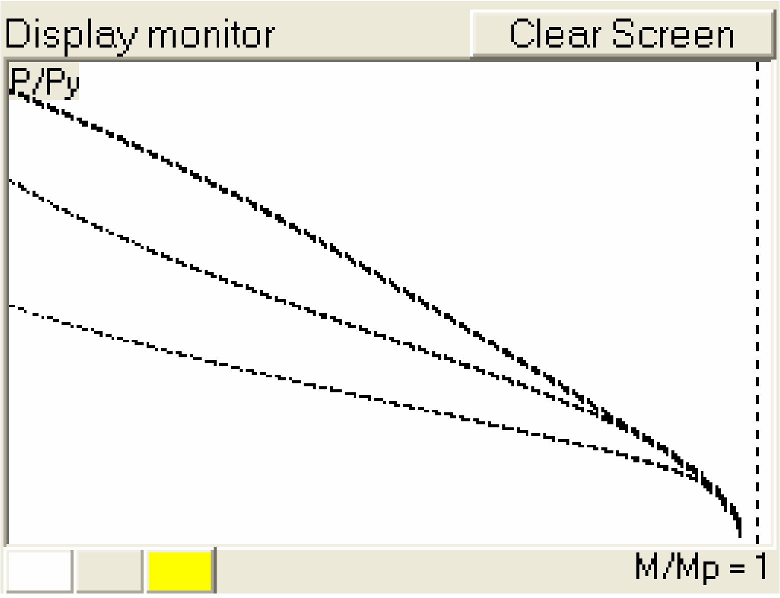


Fig 8.10 Display monitor

9. Results

The results for slenderness equal to zero, 20,40,60,80,100 and 120 are shown in fig 9.1 to fog 9.7. *Note that in Slenderness equal to zero the interaction curve is the same for capacity curve.* The results are for default values.

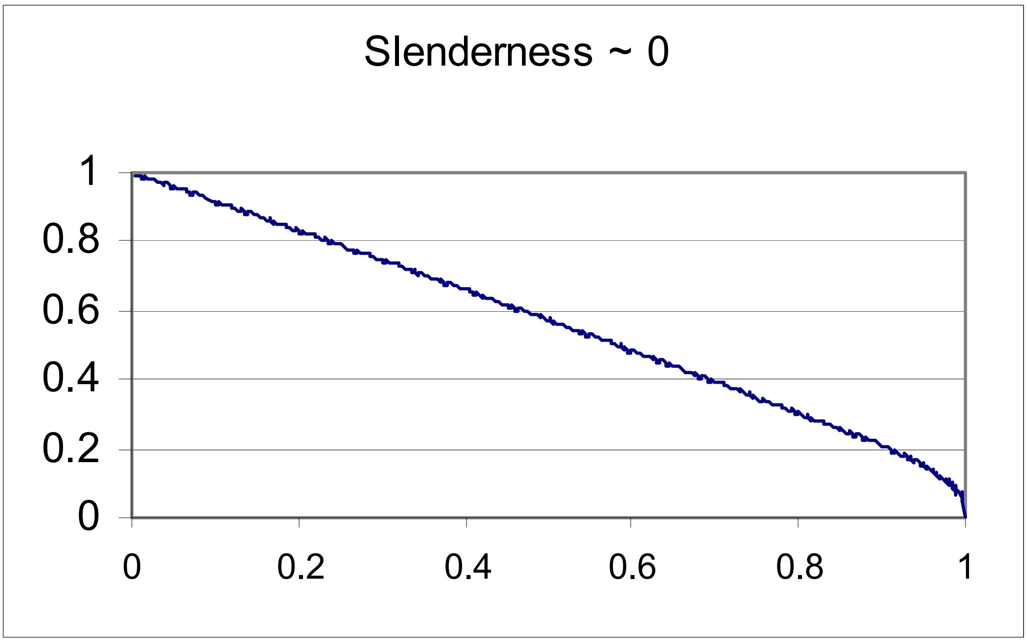


Fig 9.1

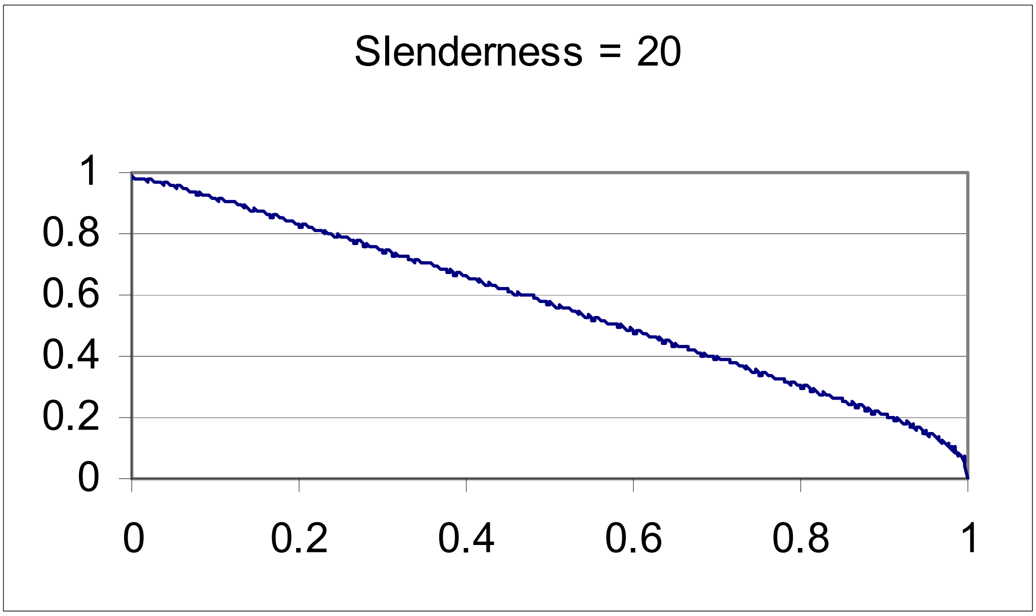


Fig 9.2

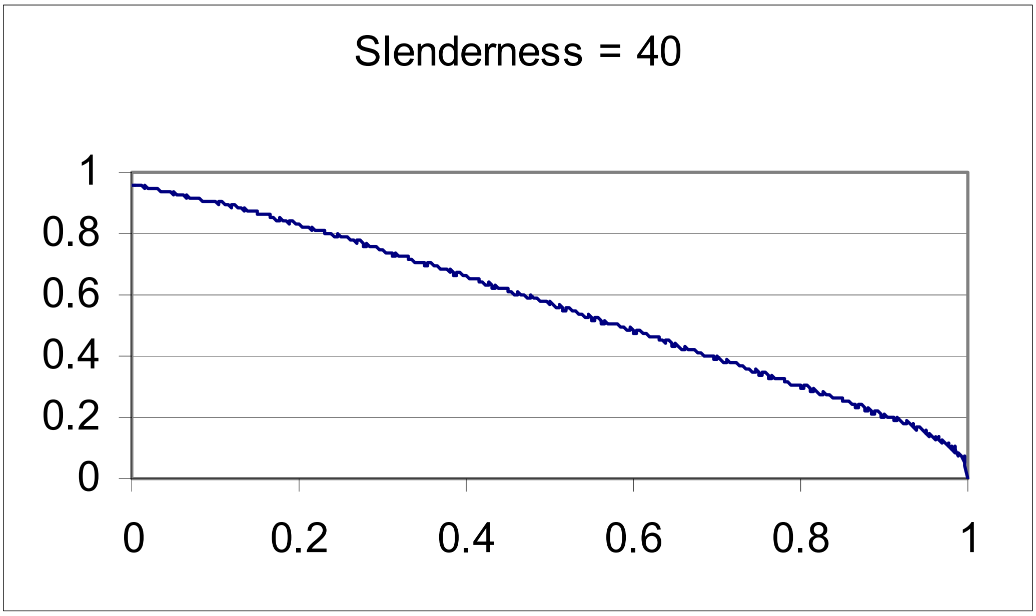

Slenderness = 40
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1

Fig 9.3

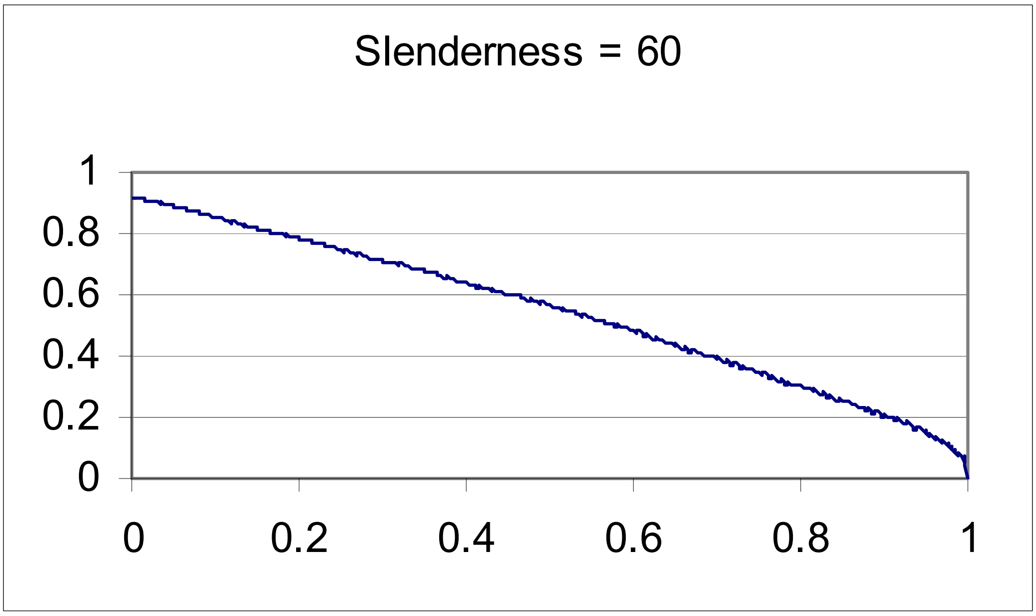

Slenderness = 60
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1

Fig 9.4

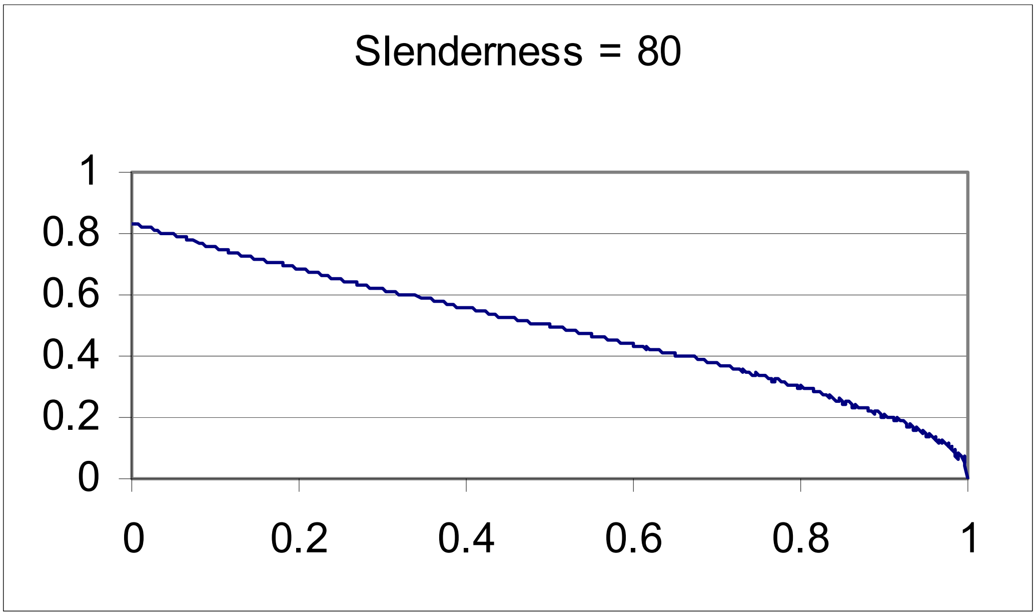

Slenderness = 80
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1

Fig 9.5

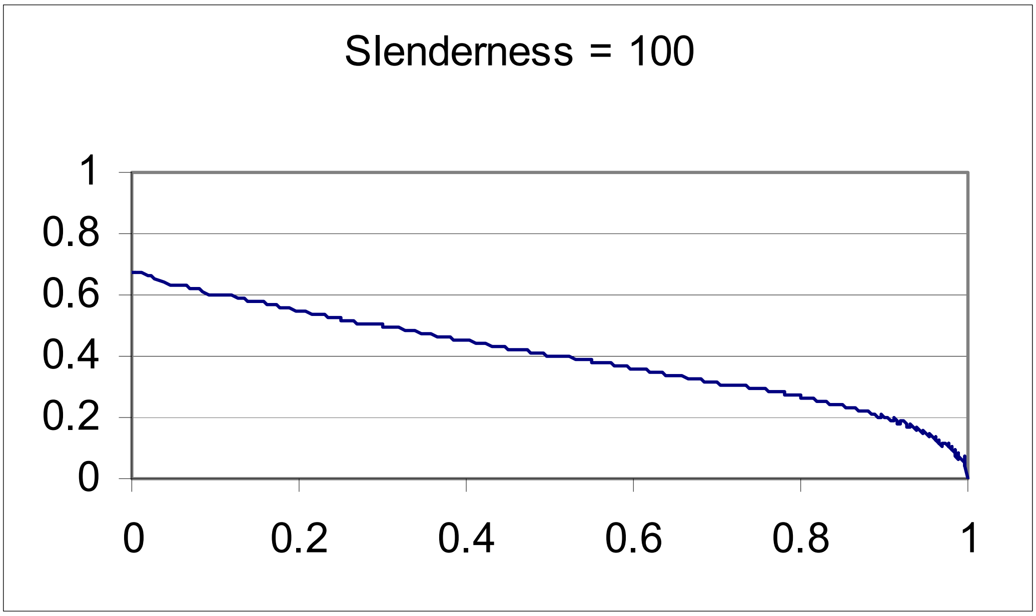

Slenderness = 100
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1

Fig 9.6

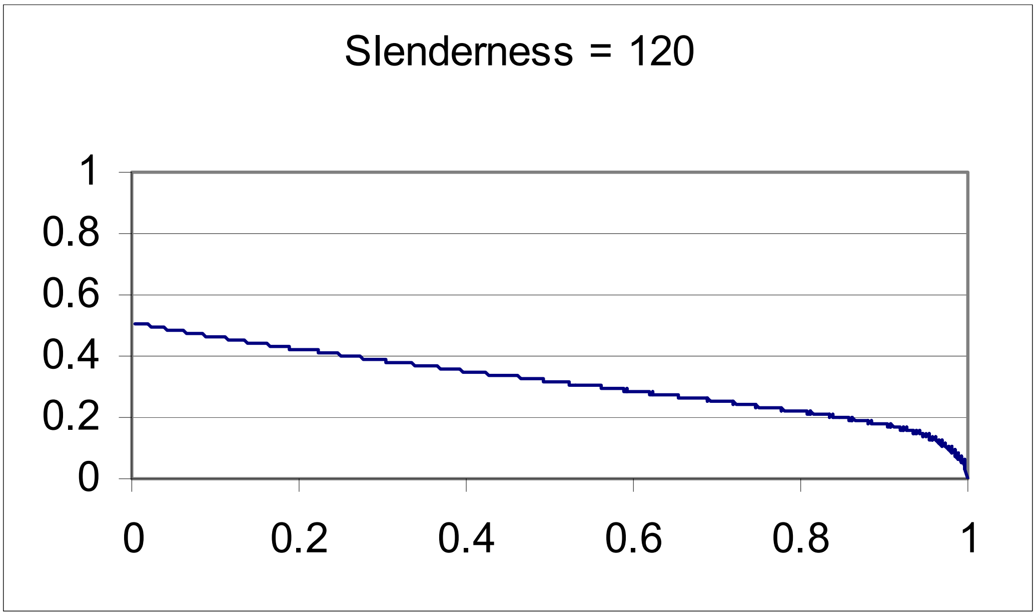


Fig 9.7